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As scientific and engineering problems increase in complexity, there is a corresponding need to automate a greater portion of the solution process. Adaptive methods are capable of providing many of the decisions that arise when solving problems involving partial differential equations. When automatic mesh generation, refinement and coarsening is combined with method-order variation, it is possible to construct methods having spectral convergence rates. Accuracy and solution reliability are verified by asymptotically correct estimates of discretization errors. With parallel computation becoming more widespread and necessary, it is important to consider parallel adaptive strategies. Much more difficult to parallelize than traditional algorithms, adaptive strategies necessarily require dynamic processor scheduling because of load changes introduced by periodic solution enrichment.

Focusing on three-dimensional transient problems for parabolic and hyperbolic systems, the goal of this research is the continued development of adaptive and parallel solution strategies and software. Our concentration will be on efficient hp- and hpr-refinement, where mesh modifications and order variations occur simultaneously, and distributed-memory MIMD computation. Temporal integration will utilize local refinement, where temporal steps and orders are spatially dependent, with both explicit and implicit schemes. Processor load balancing will occur through incremental migration strategies that have low unit cost and are appropriate for use with dynamic scheduling and adaptive processes. Parallel mesh generation and mesh enrichment procedures will complete the development of an adaptive software system that will be capable of addressing some of the most difficult and challenging problems arising in practice.

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PARALLEL ADAPTIVE TECHNIQUES FOR TRANSIENT PARTIAL DIFFERENTIAL EQUATIONS

FINAL REPORT

Joseph E. Flaherty and Mark S. Shephard

U.S. Army Research Office

Contract Number DAAL 03-91-G-0215

Scientific Computation Research Center
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1. Forward

As scientific and engineering problems increase in complexity, there is a corresponding need to automate a greater portion of the solution process. Adaptive methods can provide many of the decisions that arise when solving problems involving partial differential equations. When automatic mesh generation, refinement and coarsening is combined with method-order variation, it is possible to construct methods having spectral convergence rates. Accuracy and solution reliability are verified by asymptotically correct estimates of discretization errors. With parallel computation becoming more widespread and necessary, it is important to consider parallel adaptive strategies. Much more difficult to parallelize than traditional algorithms, adaptive strategies necessarily require dynamic processor scheduling because of load changes introduced by periodic solution enrichment.

Focusing on three-dimensional transient problems and massively-parallel distributed-memory MIMD computation, we developed a software framework that unifies mesh generation, mesh modification, and dynamic load balance. We concentrated on efficient hp- and hpr-refinement, where mesh modifications and order variations occur simultaneously. Processor load balancing occurs through incremental migration strategies that have low unit cost and are appropriate for use with dynamic scheduling and adaptive processes. Parallel mesh enrichment procedures use edge-based refinement and collapsing techniques. Applications are being considered in compressible fluid dynamics.

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2. Summary of Results

2.1. Statement of the Problem

The basic computational tools needed for the reliable solution of complex scientific and engineering problems on arbitrary three-dimensional domains are becoming available. Adaptive techniques that provide a controlled level of discretization error are maturing and procedures for the automatic discretization of generic spatial domains have been developed. With continued growth in computational power, it is becoming possible to combine solution techniques, adaptive strategies, geometric modeling procedures, and automatic mesh generation techniques to create a powerful and flexible solution environment for the solution of partial differential equations.

Parallel computation is becoming increasingly important for the solution of the realistic problems that are being addressed in modern engineering analysis. The goal of this research effort is the unification of parallel computation and automated adaptive techniques. Not only will this software provide a capability for solving previously intractable three-dimensional problems, but it will automate many of the decision processes and, thereby, give a scientist or engineer more time to concentrate on the physical, chemical, or biological aspects of the problem at hand.

During the course of this contract with the U. S. Army Research Office, we concentrated on (i) developing high-order adaptive strategies that combine order variation and mesh refinement for transient and steady systems, (ii) developing parallel adaptive strategies and data structures for execution on massively parallel computers, (iii) developing a hierarchical mesh database that can be used to solve three-dimensional problems on arbitrarily complex domains using serial or parallel computers, and (iv) applying these methods to problems of practical interest. Additional details regarding these accomplishments follow in Section 2.2. Scientific presentations, publications, and participating personnel appear in Sections 3-5, respectively.

2.2. Key Results

Brief descriptions of our important results and findings follow. Topics are listed in approximate chronological order.

2.2.1. Adaptive hp-Refinement and Error Estimation

Using our variant of the singly implicit Runge-Kutta (SIRK) method [19, 43], we developed the first adaptive hp-refinement finite element strategy for parabolic systems [13, 19, 43]. The two-dimensional software uses a finite quadtree structure to manage both mesh generation and refinement. Efficient estimates of spatial discretization errors are obtained using p-refinement to create and solve local problems having forcing proportional to elemental or edge residuals of the finite element solutions. Error estimates computed in this manner converge in energy at the correct rate to the actual finite element spatial errors under h-refinement [11]. These results are robust in time; hence, temporal variation of spatial errors may be neglected. In typical applications [13, 43], this reduces the time required to obtain an error estimate by approximately thirty percent.

Our variant of the SIRK method offers several advantages when used with hp-refinement [19, 43]. SIRKs have high-stage order, A-stability for orders one through eight, L-stability for orders one through six and eight, efficiencies close to those of

backward difference software, embedded error estimates, and a locality that is suitable for parallel computation.

Spatial error estimates obtained by p-refinement [11, 13] have a limited range of applicability when applied to singularly-perturbed parabolic and elliptic systems. Furthermore, solutions obtained by symmetric Galerkin techniques oscillate unless the computational mesh is appropriately dense within boundary and interior layers. Using singular-perturbation theory, Adjerid et al. [38] developed a strategy for stabilizing the solution of singular-perturbation problems that requires changing the quadrature rule used to evaluate inner products. They describe several rules and, among other things, show that Radau quadrature produces stable solutions of convection-diffusion systems and Lobatto quadrature does likewise for reaction-diffusion problems. With a SIRK temporal integrator, the Radau and other quadrature-related procedures were used to solve difficult nonlinear problems, including one involving shear-band formation.

Biswas et al. [25] and Devine et al. [26, 40] developed superior solution limiting and error estimation schemes for a high-order local finite element technique. When used to solve hyperbolic systems of conservation laws, the new limiting strategies preserve an essentially non-oscillatory behavior near discontinuities while maintaining a high order of accuracy in smooth regions. Following the ideas of Adjerid et al. [38], spatial error estimates utilize a p-refinement approach with superconvergence at Radau points to compute efficient and (apparently) asymptotically correct results [25, 40].

2.2.2. Mesh Data Structures and Operators

The data structures used in a parallel adaptive finite element solver provide fast query and update of partition boundary information. The necessary queries are (i) adjacency information for entities located on more than one partition, (ii) number and list of adjacent processors given an entity type adjacency, (iii) list of entities on partition boundary given an adjacent processor and entity type, and (iv) list of scatter and gather maps of nodes on the partition boundary.

Besides the queries, update procedures must be available to the refinement/coarsening and element migration/load balancing components of the parallel finite element solver. The updates should enable entities to be inserted into or deleted from the partition boundary within constant time or at most time proportional to the number of adjacent processors.

To implement these fast query and update routines, the topological *entity hierarchy* data structures [47] which provide a two-way link between the mesh entities of consecutive order, i.e., regions, faces, edges, and vertices, are used. From this hierarchy, any entity adjacency relationship can be derived by local traversals. The entities on the partition boundary are augmented with *links* which point to the location of the corresponding entity on the neighboring processor.

These inter-processor links are then maintained in a doubly linked list with a processor *id* node as the header. From these structures, partition boundary entity insertion/deletion can be made in constant time. The entities neighboring a processor can also be traversed by starting at the header node given by processor and following the doubly link list.

Each partition boundary entity can have attached to it either the *complete* or the *minimal* set of inter-processor links. In the complete set, all the boundary entities store the location of the corresponding duplicate entity. Since the lower entities inherit the higher level entity adjacency, it is possible to eliminate the inter-processor links whose adjacency can be derived from higher level entities. This minimal link representation has the advantage of reducing the storage needed to maintain the partition boundary entities. However, the minimal representation has the disadvantage of complicating the link update procedures when element migrations are performed. Therefore, a switching mechanism is used to allow both representations to be used disjointly.

2.2.3. Parallel Mesh Refinement and Coarsening

The mesh-level adaptive scheme combines coarsening, refinement, and triangulation optimization using local retriangulations [48]. The coarsening step is based on an edge collapsing technique. A mesh edge is collapsed by deleting all mesh regions connected to one end vertex and connecting the faces of the polyhedral cavity to the other end vertex. Edge collapsing is not always possible for because (i) it may lead to local topological invalidity of the triangulation and (ii) it can lead to the creation of invalid elements. It does not require storage of any history information and is, therefore, not dependent on the refinement procedure.

Refinement makes use of subdivision patterns. All possible subdivision patterns have been considered and implemented to allow for speed and annihilate possible over-refinement. If the bounding face of a mesh region to be subdivided with two and only two marked edges is already triangulated, the template for that region must be able to match the face triangulation. Since there are a priori two ways to triangulate a face with two marked edges, any pattern which has N faces with two and only two marked edges needs 2^N templates.

Triangulation optimization is necessary to prevent triangulation quality degradation. It is particularly important when the *snapping* of refinement vertices on curved model boundaries can potentially create invalid or poorly shaped elements. The idea is to iteratively consider the retriangulation of simple and well defined polyhedra. The optimization procedure builds upon the coupling of edge removal and its dual, multi-face removal. Both techniques can be seen as tools to retriangulate simple polyhedra. Edge removal consists of deleting all mesh regions connected to a mesh edge and retriangulating the space in the resulting polyhedral cavity without recreating the mesh edge to be removed. Multi-face removal is the reverse process of edge removal. At this point, it is worthwhile to note that edge collapsing is also a form of local retriangulation that removes a mesh vertex. Then, coarsening and triangulation optimization can be treated in a similar way.

Since refinement uses templates, its parallelization presents no difficulty. First, mesh faces on the partition boundary are triangulated. Triangulation compatibility is implicitly guaranteed by the fact that duplicate faces have the same orientation. All minimal inter-processor links (face and edge level) need to be updated. Then, mesh regions are triangulated without communication involved. The challenge resides in the efficient parallelization of the coarsening and triangulation optimization steps. An efficient way to retriangulate polyhedra in parallel can be decomposed in three steps: (i) retriangulate polyhedra which are fully interior to the partition, (ii) shift the partition boundary using element migration techniques, and (iii) retriangulate those polyhedra that are now fully accessible due to the shift.

In the case of triangulation optimization which is iterative by nature, shifting the partition boundary always in the same direction will quickly create load imbalance. If one iteration makes elements to be migrated in one direction (from processor i to processor j), the next iteration will reverse the flow (from processor j to processor i). In other words, the partition boundary oscillates from one iteration to the next.

2.2.4. Parallel Procedures

Our initial parallel solution procedures focused on adaptive techniques for shared-memory computers where the need for synchronization can be reduced by ''coloring'' elements, faces, or edges that have independent basis support and, hence, may be assembled and solved in parallel [1, 2, 6, 18, 21, 42]. Recent endeavors have involved distributed-memory computation on single- and multiple-instruction systems. Processor scheduling and load balancing must be dynamic since adaptivity will upset a balanced computation. Partitioning must, furthermore, have a low cost per solution step since several adaptive enrichments are needed during the course of a computation. Thus, we utilize incremental migration techniques where finite elements are interchanged between processors as the solution evolves and processor demands change.

Tiling [16, 25, 26, 40, 41, 46] is a modification of a global load balancing technique of Leiss and Reddy who balanced local loading within overlapping processor neighborhoods. Each processor is the center of a neighborhood and loading is balanced within neighborhoods using local performance measurements. Work is exchanged between processors in the neighborhood by migrating elements to processors having neighboring elements.

Using tiling as a framework, we developed three-dimensional migration tools that are intimately related to the SCOREC hierarchical mesh database. Capabilities are available for automatic mesh generation; parallel partitioning using recursive coordinate, inertial, or spectral bisection; automatic mesh refinement and coarsening; and migration. All partitioning and migration operators are capable of handling arbitrary combinations of h-, p-, and hp-refinement. Work between migration steps is calculated and possible neighborhood workload requests are assembled into a forest of trees. Processor loading is balanced on each tree and the procedure can be iterated to achieve a more global balance [32, 37, 41]. Convergence has typically been rapid and an acceptable balance requires a handful of iterations.

Example 1. The migration/balancing procedures have been applied to several problems. As an example, we partition the flow region surrounding a tandem helicopter (cf. Figure 1a) using the moment of inertia strategy. A planar slice through the three-dimensional region consisting of 94,069 tetrahedral elements is shown in Figure 1b. Mesh refinement was performed on one processor of a 16-processor IBM SP/1 computer. This introduces an load imbalance which is restored using migration. The migration procedure restores balance within a half-dozen iterations (cf. Figure 1c). The communications cost is measured by the maximum ratio of the number of element faces on a partion boundary to the total number of faces with that partition. This ratio equilibrates at approximately 20% after iterative migration is applied (cf. Figure 1d). While acceptable relative to other procedures, we believe that this ratio can be further reduced.

The three-dimensional partitioning and migration framework is generic and, e.g., does not utilize the underlying octree structure of the mesh. Such usage could potentially improve the partitioning. We, thus, are investigating a simple tree-partitioning strategy

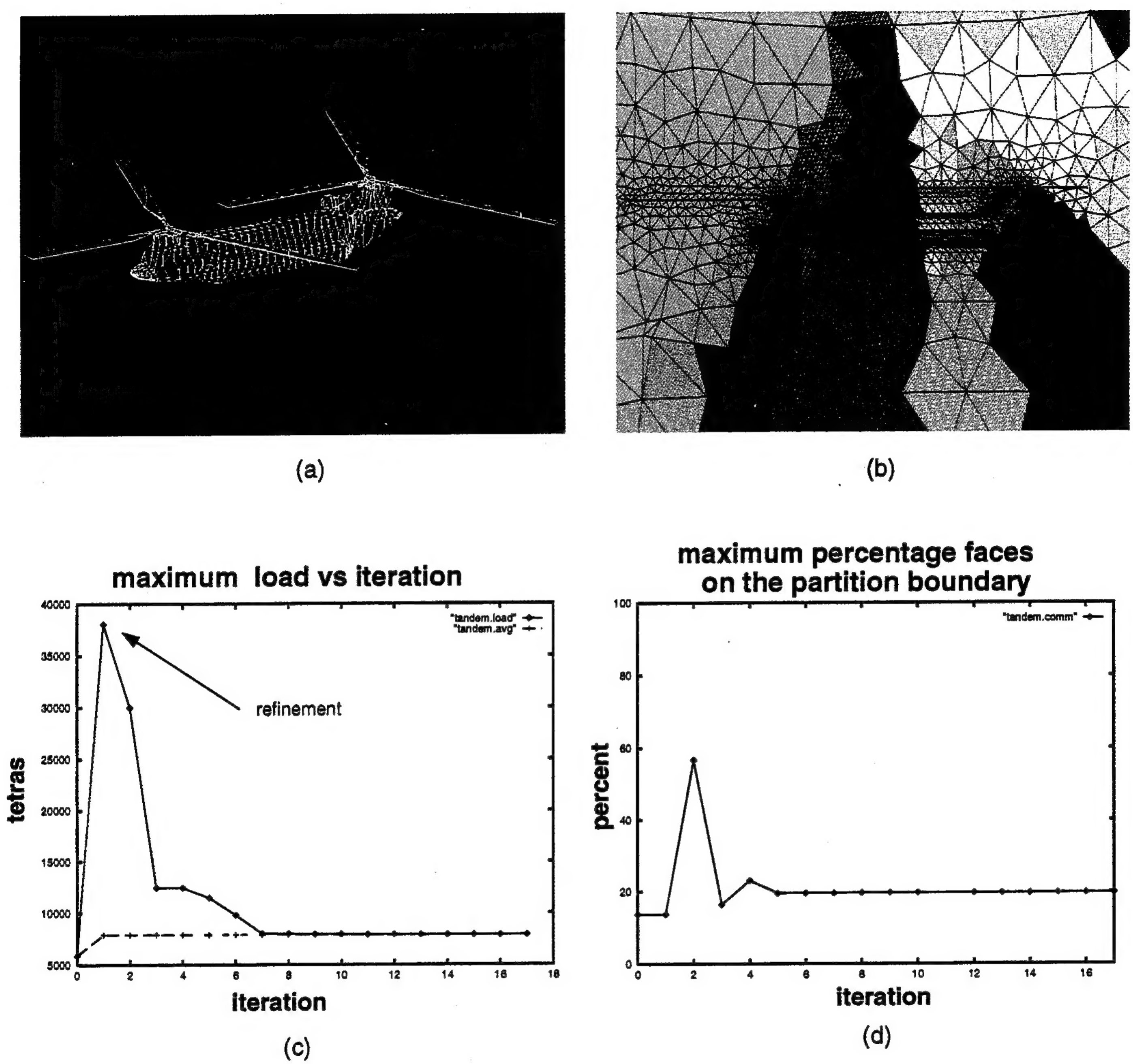

Figure 1. Tandem helicopter, surface mesh (a); two-dimensional slice of the moment-of-inertia partitioning of a mesh of 94,069 tetrahedral elements on a 16-processor computer (b) ; load imbalance introduced on one processor and its restoration using iterative migration (c) ; and communications cost as measured by the maximum percentage of element faces on partition boundaries (d).

that (i) determines cost metrics for all subtrees and (ii) partitions the octree according to these metrics [30, 40, 41]. The partitions consist of octants that are each the root of a subtree and are determined by a truncated depth first search.

Example 2. The tree-based partitioning strategy has been applied to a series of six finite-octree generated meshes [40] involving flow solutions about airplanes, helicopters, wings, and cones. The number of elements in each mesh ranged from 16,000 to 293,000 with and average of 174,000. Partition quality is measured as the percent of element faces on inter-partition boundaries relative to the total number of faces of the mesh. In Figure 2, we display these percentages as a function of the optimal partition size. The variance between partition costs is as small as the cost of a leaf octant in all cases.

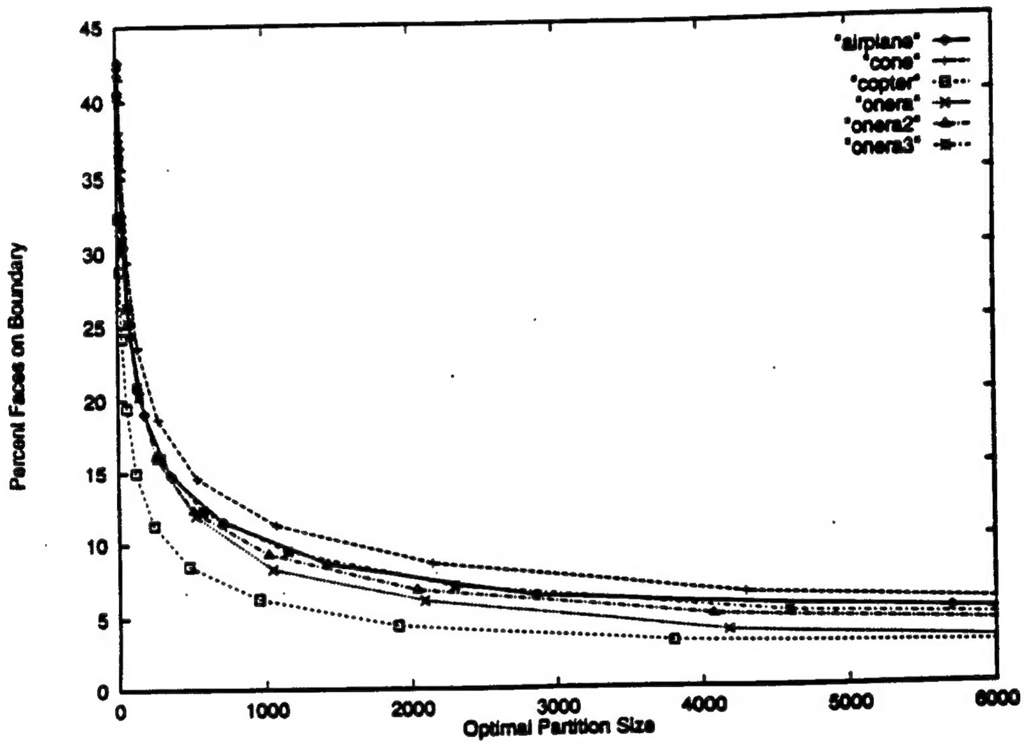

Figure 2. Performance of the octree partitioning algorithm on six meshes.

The performance metric shown in Figure 2 is a measure of the total surface area that partitions have in common. Smaller values require less communication relative to the local volume of data. The interface proportion is less than 12% when the partition size exceeds 1000 elements and drops to below 9% for partitions of 2000 elements or more. This performance is comparable to recursive spectral bisection but has a linear instead of a quadratic time complexity. Comparisons between the data shown in Figure 1d with those of Figure 2 can be made by dividing the former results by two.

While there is a great deal more work to develop production systems for adaptive and parallel computation, the framework developed here provides an effective an efficient platform for further research and implementation. The software is simultaneously being used to solve difficult problems involving rotorcraft flows [32].

3. Interactions

Invited lectures and other interactions by personnel involved with research supported by this contract follow for the past two years.

1. J.E. Flaherty presented lectures "Adaptive Methods for Time-Dependent Partial Differential Equations. Part I: Basic Strategies and Error Estimation" and "Part II: High-Order Methods and Parallel Computation," *Dutch Numerical Analysis Seminar,* Woudschoten Conference Center, Zeist, October 5-7, 1992. This is a national symposium attended by university faculty, industrial scientists, and graduate students throughout the Netherlands. It features three keynote lecturers who, in addition to Flaherty, were Charles Chui (Texas A & M University) Claes Johnson (Chalmers Technological University).

2. M.S. Shephard lectured on "Issues and Advances in Automatic 3-D Mesh Generation with Emphasis on Octree-Based Techniques," and "Adaptive Unsteady Aerodynamics Using the Time-Discontinuous GLS Finite Element Method." *AGARD Lecture Series on Unstructured Grid Methods for Advection Dominated Flows.* NASA AMES Research Laboratories, Mountain View, CA, October 2, 1992.

3. M.S. Shephard lectured on "Finite Element Model Generation, Integration of Finite Element Modeling with Engineering Design and Ensuring the Reliability of Finite Element Results." University of Pennsylvania, Philadelphia, October 20, 1992.

4. J.E. Flaherty lectured on "High-Order Adaptive Methods for Time-Dependent Partial Differential Equations," Washington University, St. Louis, October 29, 1992.

5. J.E. Flaherty lectured on "Coloring Procedures for Finite Element Computation on Shared-Memory Computers," *Symposium on Adaptive, Multilevel, and Hierarchical Computational Strategies,* ASME Winter Annual Meeting, Anaheim, November 8-13, 1992.

6. M.S. Shephard lectured on "Idealization Control for the Analysis of Composite Materials." *Symposium on High-Performance Computing for Flight Vehicles*, Washington, December 8, 1992.

7. M.S. Shephard lectured on "SCOREC Research Activities and Parallel Processing," IBM, Kingston, NY, December 18, 1992.

8. J.E. Flaherty lectured on "Adaptive and Parallel High-Order Methods for Conservation Laws," Tulane University, New Orleans, January 19, 1993.

9. J.E. Flaherty lectured on "Adaptive and Parallel High-Order Methods for Conservation Laws," Mississippi State University, Mississippi State, January 22, 1993.

10. J.E. Flaherty lectured on "Adaptive and Parallel High-Order Methods for Conservation Laws," University of Texas, Austin, March 13-22, 1993.

11. J.E. Flaherty lectured on "Adaptive and Parallel High-Order Methods for Conservation Laws," University of Kentucky, Lexington, April 21, 1993.

12. M.S. Shephard lectured on "Reliable Automated Engineering Analysis in an Integrated Design Environment." *NAFEMS 4th Int. Conf. on Quality Assurance and Standards in Finite Element and Associated Technologies,* Brighton, UK, May 27,

1993.

13. J.E. Flaherty, with Julian Cole and Donald Drew of Rensselaer and Bernard Matkowsky of Northwestern University and Lu Ting of New York University, organized and ran a workshop on *Perturbation Methods in Physical Mathematics* at Rensselaer, June 23-26, 1993. The workshop honored Joseph B. Keller and recognized his seventieth birthday. More than one hundred scientists attended the symposium. Flaherty lectured on ''Adaptive Numerical Procedures for Singularly Perturbed Partial Differential Equations.'' Written proceedings of this workshop will be published as a special issue of *SIAM Journal of Applied Mathematics.*

14. J.E. Flaherty, with Ivo Babuska of the University of Maryland, William Henshaw of IBM, John Hopcroft of Cornell University, Joseph Oliger of Stanford University, and Tayfun Tezduyar of the University of Minnesota organized a summer program on *Modeling, Mesh Generation, and Adaptive Numerical Methods for Partial Differential Equations* at the Institute for Mathematics and its Applications of the University of Minnesota, Minneapolis, July 6-23, 1993. Shephard presented (3) keynote lectures on ''The Automatic Generation of Valid Finite Element Meshes for General 3-D Geometric Domains,'' ''The Integration of Automatic Mesh Generation with Geometric Modeling,'' and ''Mesh Control Functions and Automated, Adaptive Analysis with Automatic Mesh Generator.'' Flaherty lectured on ''Adaptive and Parallel High-Order Methods for Conservation Laws.''

15. M.S.Shephard lectured on ''The Automatic Generation of Finite Element Models and Integration with Geometric Modeling,'' Indian Institute of Technology, Madras, August 5, 1993.

16. M.S. Shephard lectured on ''Automated Adaptive Analysis for Engineering Design,'' Indian Institute of Technology, Madras, India, August 6, 1993.

17. J.E. Flaherty lectured on ''Computers: Small and Large, Serial and Parallel,'' ''An Introduction to the Parallel Solution of Finite Element Problems,'' ''Parallel Solution Procedures on Shared and Distributed Memory Computers,'' and ''Adaptive and Parallel Finite Element Methods for Conservation Laws,'' Summer workshop on *Frontiers in Finite Element Technology,* Győr, August 16-19, 1993. This Hungarian-sponsored symposium was attended by faculty and graduate students from former iron-curtain countries. The three keynote lecturers, Flaherty, Ivo Babuska of the University of Maryland, and Barna Szabo of Washington University, each presented a sequence of four talks.

18. M.S. Shephard lectured on ''Problems in Automatic Mesh Generation for Unsteady Transonic CFD,'' *Fifth Int. Workshop on Dynamics and Aeroelastic Stability Modeling of Rotorcraft Systems*, Troy, October 18, 1993.

19. M.S. Shephard lectured on ''Finite Element Model Generation, Integration of Finite Element Modeling with Engineering Design and Ensuring the Reliability of Finite Element Results.'' University of Pennsylvania, Philadelphia, October 19, 1993.

20. M.S. Shephard lectured on ''Parallel Adaptive Mesh Refinements and Redistributions on Distributed Memory Machines,'' *Symposium on Parallel Finite Element Computations,* Supercomputer Institute, Minneapolis, October 26, 1993.

21 M.S. Shephard lectured on ''Parallel Finite Octree Automatic Mesh Generation,'' NAS, NASA Ames Research Center, Mountain View, CA, October 28, 1993.

22 M.S. Shephard lectured on ''Finite Octree Automatic Mesh Generation and its Parallelization,'' Argonne National Laboratory, Argonne, IL, December 9, 1993.

23 M.S. Shephard lectured on ''Automated Finite Element Analysis Techniques for Rotorcraft CFD,'' Boeing Aircraft, Philadelphia, PA, February 22, 1994.

24. J.E. Flaherty lectured on ''Adaptive and Parallel High-Order Methods for Conservation Laws'' at the University of Toronto, February 25, 1994.

25. J.E. Flaherty attended the *Workshop on Basic Phenomena in Plasticity* at Carnegie Mellon University, March 17-19, 1994. He lectured on ''Adaptive Methods for Partial Differential Equations with Application to Shear Band Formation.''

26. M.S. Shephard lectured on ''Finite Element Computations,'' *Super 94 Conference,* LSU, Baton Rouge, LA, April 26, 1994.

27. J.E. Flaherty lectured on ''Adaptive Methods for Parabolic Partial Differential Equations'' at the University of Buffalo, May 5, 1994.

28. M.S. Shephard lectured on ''Parallel Automated Adaptive Finite Element Methods,'' Twelfth U.S. National Congress on Applied Mechanics, Seattle, Washington, June 30, 1994.

29. J.E. Flaherty lectured on ''Parallel Adaptive Load-Balancing Schemes for Three-Dimensional Conservation Laws,'' at the symposium on *Adaptive and Parallel Methods for CFD, 14th IMACS World Congress on Computation and Applied Mathematics,* 11-15 July 1994, Atlanta.

30. M.S. Shephard lectured on ''Parallel Finite Element Analysis", Third World Congress on Computational Mechanics, Int. Assoc. for Computational Mechanics, OVTA, Makuhari, Chiba, Japan, August 1, 1994.

31. M.S. Shephard lectured on ''Parallel Automatic Three-Dimensional Mesh Generation,'' Third World Congress on Computational Mechanics, Int. Assoc. for Computational Mechanics, OVTA, Makuhari, Chiba, Japan, August 2, 1994.

32. M.S. Shephard lectured on ''Parallel Adaptive Finite Element Analysis of Flows on Distributed Memory Computers,'' Recent Developments in Finite Element Analysis, Palo Alto, CA, September 21, 1994.

33. M.S. Shephard lectured on ''Parallel Adaptive hp-FEM for Structural Acoustics Applications,'' Office of Naval Research Meeting on Parallel Solution Methods in Structural Acoustics, University of Maryland, College Park, MD, Sept. 23, 1994.

4. M.S. and Ph.D. Degrees Awarded

Those graduate students who were supported by this contract and received their M.S. or Ph.D. degrees during the course of this contract follow.

1. Rupak Biswas, *Parallel and Adaptive Methods for Hyperbolic Partial Differential Systems,* 1991, Ph.D., Computer Science, Rensselaer Polytechnic Institute. Currently: Postdoctoral Fellow, Research Institute for Advanced Computer Science, NASA Ames Research Center. (Working with R. Strawn on ARO rotorcraft problems.)

2. Messaoud Benantar, *Parallel and Adaptive Algorithms for Elliptic Partial Differential Systems,* 1992, Ph.D., Computer Science, Rensselaer Polytechnic Institute. Currently: IBM, MVS System Design.

3. Hugues L. de Cougny, *Automatic Generation of Geometric Triangulations based on Octree/Delauney Techniques,* 1993, M.S., Civil and Environmental Engineering, Rensselaer Polytechnic Institute. Currently: Research Engineer, SCOREC, and Ph.D. Student, Rensselaer Polytechnic Institute.

4. J. Michael Coyle, a mathematician at Benét Laboratories, received his Ph.D. in Mathematics at Rensselaer in 1993 under Flaherty's direction. His dissertation title was *An Hr-Refinement Finite Element Method for Systems of Parabolic Partial Differential Equations with Stability Analyses for Mesh Movement.*

5. Karen Devine received her Ph.D. in Computer Science in 1994 under Flaherty's direction. Her dissertation title was *An Adaptive HP-Finite Element Method with Dynamic Load Balancing for the Solution of Hyperbolic Conservation Laws on Massively Parallel Computers.* Her research stipend was provided by Sandia National Laboratories, but the subject of her dissertation coincided with this contract and Flaherty's time was supported by ARO. Currently: Postdoctoral Fellow, Massively Parallel Computation Research Laboratory, Sandia National Laboratories.

5. Publications and Manuscripts

Papers published or in press and submitted manuscripts on research supported by this contract follow.

Publications

1. M. Benantar, R. Biswas, and J.E. Flaherty, "Adaptive Methods and Parallel Computation for Partial Differential Equations," *Trans. Eighth Army Conf. on Applied Maths. and Comput., ARO Report 91-1,* U.S. Army Research Office, Research Triangle Park, 531-552, 1991.

2. M. Benantar, R. Biswas, and J.E. Flaherty, "Advances in Adaptive Parallel Processing for Field Applications," *IEEE Trans. Magnetics,* **27** (1991), 3768-3773.

3. P.L. Baehmann, M.S. Shephard, and J.E. Flaherty, "A Posteriori Error Estimation for Triangular and Tetrahedral Quadratic Elements Using Interior Residuals," *Int. J. Numer. Meths. Engng.,* **34** (1992), 979-996.

4 P.S. Donzelli, R.L. Spilker, P. L. Baehmann, Q. Niu and M. S. Shephard, "Automated Adaptive Analysis of the Biphasic Equations for Soft Tissue Mechanics Using A Posterior Error Indicators," *IJNME,* **34,** (1992), 1015-1033.

5. P.K. Moore and J.E. Flaherty, "Adaptive Local Overlapping Grid Methods for Parabolic Systems in Two Space Dimensions," *J. Comp. Phys.,* **98** (1992), 54-63.

6. J.E. Flaherty, M. Benantar, R. Biswas, and P.K. Moore, "Symbolic and Parallel Adaptive Methods for Partial Differential Equations," Chap. 12 in B.R. Donald, D. Kapur, and J.L. Mundy, Eds. *Symbolic and Numerical Computation for Artificial Intelligence,* Academic Press, London, 277-302, 1992.

7. J.E. Flaherty and Y.J. Wang, "Experiments with an Adaptive h-, p-, and r-Refinement Finite Element Method for Parabolic Systems," in G.D. Byrne and W.E. Schiesser, Eds., *Recent Developments in Numerical Methods and Software for ODEs/DAEs/PDEs,* World Scientific, 55-80 Singapore, 1992.

8. Q. Niu and M.S. Shephard, "Superconvergent Extraction Techniques for Finite Element Analysis," *IJNME,* **36,** (1993), 811-836.

9. D.C. Arney, R. Biswas, and J.E. Flaherty, "An Adaptive Mesh Moving and Refinement Procedure for One-Dimensional Conservation Laws," *Appl. Numer. Math.,* **11** (1993), 259-282.

10. B.E. Webster, M.S. Shephard, Z. Rusak and J.E. Flaherty, "Unsteady Compressible Airfoil Aerodynamics Using The Time-Discontinuous GLS Finite Element Method," AIAA 31st Aerospace Sciences Meeting, American Institute for Aeronautics and Astronautics, 93-0339, January 11-14, 1993, Reno, Nevada.

11. S. Adjerid, J.E. Flaherty, and Y.J. Wang, "A Posteriori Error Estimation with Finite Element Methods of Lines for One-Dimensional Parabolic Systems," *Numer. Math.,* **65** (1993), 1-21.

12. M. S. Shephard, "Status of 3-D Mesh Generation and an Adaptive Analysis Example," Advanced Study Institute on Computational Methods for Engineering Analysis and Design, Indian Institute of Technology, Madras, India, 3.1-3.23, 1993.

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